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CHARACTERISTICS OF TRANSVERSE ACOUSTIC WAVES IN THE MK 12 MOTOR--ETC(U)
MAY 76 H B MATHES, C J BICKER

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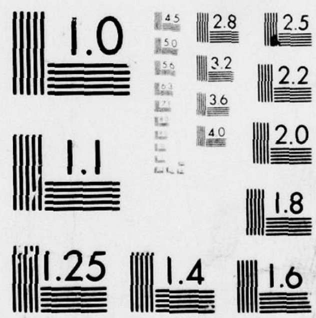
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# CHARACTERISTICS OF TRANSVERSE ACOUSTIC WAVES IN THE MK 12 MOTOR

by

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C. J. Bicker
Research Department

May 1976





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Results are presented of digital computer predictions of acoustic resonance characteristics of the Mk 12 solid rocket motor cavity. Transverse acoustic wave solutions were obtained for internal geometries representing six different times during burn. A plot of acoustic mode frequency as a function of web fraction burned, isometric plots of pressure distribution for selected modes at a given web burn, a table of modal frequencies with descriptions of wave structure, and a graphical comparison of predicted oscillation frequencies versus measured vibrational frequencies are presented.

Comments are offered regarding the effect of high frequency pressure transducer placement on acoustic wave measurement quality.

Correlation of acoustic and vibration frequencies indicate that Mk 12 vibrations originate from combustion-generated acoustic oscillations within the motor cavity.



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## FOREWORD

This report summarizes activities relating to determination of internal acoustics of the Mk 12 motor. Work was performed by the Naval Weapons Center and sponsored by the Naval Sea Systems Commander under SEATASK 310-037-026, during the period 1 January 1975 through 26 March 1976.

This is an informal report containing interim information.

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10 May 1976

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# ABSTRACT

Results are presented of digital computer predictions of acoustic resonance characteristics of the Mk 12 solid rocket motor cavity. Transverse acoustic wave solutions were obtained for internal geometries representing six different times during burn. A plot of acoustic mode frequency as a function of web fraction burned, isometric plots of pressure distribution for selected modes at a given web burn, a table of modal frequencies with descriptions of wave structure, and a graphical comparison of predicted oscillation frequencies versus measured vibrational frequencies are presented.

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CONTENTS





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## INTRODUCTION

Experience with the Mk 12 motor in flight and in static testing indicates that vibrations, possibly caused by combustion instability, are present during the time the propellant burns. An undesirable consequence of the vibrations is occasional malfunction of the autopilot activation switch which is located in a section immediately forward of the Mk 12 motor.[1] An important means for determining if the vibrations of a rocket motor are due to combustion instability of the propellant is to compare measured structural vibration frequencies with predicted acoustic oscillation frequencies of the motor cavity. If the vibration and acoustic frequencies are similar it is probable that the vibrations are generated by combustion-gas oscillations in the motor.[2] The acoustic modes of cavities with the shape of a right circular cylinder, with no mean flow, and with uniform speed of sound in the cavity can be calculated quickly and easily using the classical equations.[3,4] Preliminary hand calculations, using the classical acoustic equations for a cylindrical form, were applied to the Mk 12 geometry as outlined in Appendix A. However, many solid rocket motor cavities, including the Mk 12, depart sufficiently from the simple cylindrical form that special techniques are required in such cases to obtain accurate predictions of acoustic modal frequencies and pressure fields. Finite element techniques, in conjunction with large high-speed digital computers, are currently being used to solve such problems. Such an approach was used to provide solutions for determining the transverse acoustic wave characteristics of the Mk 12 motor.

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[1]The malfunction problem has been alleviated by altering the switch design. However, analysis and understanding of the source of motor vibrations is considered to be an important step in improved missile performance.

[2]H. B. Mathes and E. W. Price. "Method of Determining Characteristics of Acoustic Waves in Rocket Motors," *Twelfth Aerospace Sciences Meeting*, Washington, D.C., 30 January - 1 February 1974, American Institute of Aeronautics and Astronautics, New York, Paper No. 74-199.

[3]R. D. Smith and D. F. Sprenger. "Combustion Instability in Solid Propellant Rockets", *Fourth Symposium on Combustion*, Williams & Wilkins Company, Baltimore, 1953.

[4]Naval Weapons Center. *Measurement Problems Related to Solid Rocket Combustion Instability*. China Lake, Calif., NWC, July 1968. (NWC TP 4593, publication UNCLASSIFIED.)



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## APPROACH

At NWC a finite element computer program (NASTRAN) is available which was originally developed for structural vibration analysis. It has been modified to solve pseudo-three-dimensional acoustic oscillation problems. The basic NASTRAN "3-D" acoustics program requires that at any cross-section the cavity must be cylindric symmetric, the cavity cross-section must be primarily circular, and any slots which may be present must be "narrow" relative to the cylindrical centerbore.[5] The slot width restriction is necessary due to mathematical limitations that do not allow for pressure gradients across the slots. In addition, a sinusoidal pressure variation around the axis of symmetry is required for tangential acoustic waves. Since the Mk 12 internal geometry represents a configuration that does not allow narrow slot or sinusoidal pressure variation requirements to be met, the "3-D" NASTRAN program could not be applied.

In the Mk 12 motor the frequencies of primary interest are those associated with transverse (tangential and radial) acoustic waves. In addition, the internal geometry of the motor is constant over a major portion of the grain length. These facts, therefore, made it feasible to apply a two-dimensional finite element analysis. The analysis developed by the second author of this report is a two-dimensional technique which uses portions of the standard 3-D NASTRAN program. The two-dimensional program approach is belived to be capable of providing accurate acoustic solutions for the geometry found in the Mk 12 motor.

The two-dimensional analysis assumes that the cavity boundaries are rigid. Thus possible interactions between the acoustic wave in the gas and solid portions of the motor, due to transfer of mechanical energy to the propellant or motor case, are ruled out. Such solid-gas energy transfer are probably of at least second-order importance insofar as acoustic frequency determination is concerned. Acoustic frequency determination does depend, however, on the characteristics of the gas within the cavity. In the Mk 12 there is a problem regarding the characteristic gas parameters since the propellant grain is composed of segments of two propellant compositions which have different flame temperatures and different gas compositions. Thus the temperature, average molecular weight, and specific heat ratio of gas filling the motor cavity vary with location within the cavity. In order to simplify solution of the acoustic problem, it was assumed when determining inputs

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[5]National Aeronautics and Space Administration. *The NASTRAN User's Manual*, by Caleb W. McCormick, ed. Washington, D.C., NASA, 1972. (NASA SP-222(01), publication UNCLASSIFIED.)

for the two-dimensional computer program, that gases from the two propellants were thoroughly mixed within the motor cavity. The gas properties used in the computer program, therefore, were weighted averages. The weighting was determined by the ratio of initial burning surface areas of the two propellants.

The two-dimensional NASTRAN acoustic mode program provides a set of solutions for a specific geometry of the gas-filled cavity. Since the cavity shape and dimensions change with the amount of propellant web consumed, it is necessary to solve for the acoustic modes for several internal configurations; each configuration represents a different time during burn (or a different fraction of propellant web burned). Scale drawings were prepared showing the initial configuraiton (zero web burn) at the head end of the motor. Regression of the initial surface was calculated using the appropriate burning rate data for the two propellant compositions in the motor. Five additional scale drawings of internal configurations representing 1.5, 25, 50, 75, and 100% web burn were drawn. Each drawing was divided into elements and the coordinates of the grid intersections (element corners) were used as input to the computer to define the cavity geometry. The 25% web burn grid configuration shown in Figure 1 is typical of the pattern used for the other web burns.

## RESULTS

The two-dimensional NASTRAN acoustics program provides both tabulated and graphic output. A particularly useful feature is a graphic output which provides an isometric view of the finite-element grid. This view shows the acoustic wave structure for each standing wave solution with the acoustic pressure vectorially represented at each grid point in the cavity.

In the following discussion, the isometric views are used in conjunction with descriptive terms which are derived from the structure of classical tangential acoustic waves in a simple circular cavity. In some respects this approach is awkward since the Mk 12 geometry departs sufficiently from the classical picture to have some acoustic standing wave modes which would not normally occur in a simple circular geometry. However, this approach does provide a qualitative framework around which a discussion can be structured. The description of results uses the 25% web burn as a typical example.

The first (lowest frequency) transverse mode for the 25% web burn configuration occurs at 608 Hz which appears to be a coupled cavity mode rather than a tangential mode. In the coupled-cavity mode the amplitude

and phase of the acoustic pressure are approximately equal throughout each cavity with the phase differing by 180 degrees from one cavity to the other. The relative pressure distribution in the motor cavity is vectorially depicted in Figure 2.

The second calculated mode, shown in Figure 3a, qualitatively resembles a first tangential mode in a right circular cylinder except that the phase of the acoustic pressure differs by 180 degrees across the wedges of propellant that protrude into the cavity.

Figure 3b depicts the next highest transverse wave frequency which qualitatively resembles a classical first tangential mode. The relative pressure distributions for the next higher modes are depicted in Figures 3c and 3d. Table 1 summarizes the modal frequency data for each configuration analyzed. An attempt has been made in Table 1 to classify the mode types in relation to those present in a circular cylinder. However, restrictions in the adequacy of this approach to describing acoustic wave structure in the Mk 12 geometry should be recognized. In Table 1 the term "normal" is used to designate a pressure distribution in which the pressure across the propellant wedge does not change sign. An example of this is shown in Figure 3b. In an "abnormal" tangential the phase across the propellant web differs by 180 degrees as is shown in Figure 3a.

TABLE 1. Mk 12 Computer Predicted Acoustic Wave Structures and Frequencies

| Wave structure | Fraction of web burned | | | | |
|---|---|---|---|---|---|
| | 1.5% | 25% | 50% | 75% | 100% |
| Coupled-cavity, Hz | 432 | 608 | 945 | 1,313 | ... |
| First tangential, Hz | | | | | |
| normal | 2,346 | 2,102 | 1,805 | 1,557 | 1,357 & 1,362[a] |
| abnormal | 2,115 | 1,960 | 1,811 | 2,025 | ... |
| Section tangential, Hz | | | | | |
| normal | 4,202 | 3,454 | 3,012 | 2,587 | 2,265 & 2,267[a] |
| abnormal | 4,072 | 3,578 | 3,009 | 2,781 | ... |

[a] Two frequencies are calculated due to small imperfections in the finite element grid which permit two nearly identical but distinct standing wave solutions to be obtained.



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## DISCUSSION

A comparison of calculations of tangential mode frequencies using the classical equations for a circular cavity and the finite element computer program is shown in Figure 4. The solutions tend to converge for the 100% web burn configuration, as they should, since at that time the Mk 12 cavity geometry is that of a cylinder. The difference between the two sets of calculations for first and second tangentials at burnout is mostly due to slightly different values used for the speed of sound in the gas. The majority of differences at burnout between the two types of calculations for the third tangential is due to the size of the finite element grid starting to become an error factor at the higher frequency of the third tangential.

A matter of practical importance which merits comment involves problems related to location of high frequency response pressure transducers for acoustic pressure measurement (see footnote 4). It is obvious that in order to attain effective measurements the transducer must be in the vicinity of a pressure antinode (a region of maximum acoustic pressure). A review of Figure 3 shows that optimum locations for acoustic pressure measurement of tangential waves are at the base of the propellant wedges which protrude into the motor cavity. These regions are indicated in Figure 5a by shaded areas. A qualitative analysis of tangential wave structural characteristics shows that for a configuration with protrusions into the cavity, as in the Mk 12 motor, the optimum pressure transducer locations indicated in Figure 5a allow detection of <u>any</u> order of tangential wave.

Practical difficulty with transducer placement on a motor sometimes requires a non-optimum pressure transducer location. An example of such a situation is illustrated in Figure 5b. (It should be recognized that the locations indicated will be optimum areas for all <u>even</u> numbered tangential waves but these areas will be non-optimum for those which are <u>odd</u> numbered.) The least desirable of all pressure transducer locations is in the area of the channel connecting the two cavities since that area is generally one of minimal pressure perturbation for tangential wave of any order.

## CONCLUSIONS

The relationship between vibration frequencies measured during burn using an accelerometer in the module containing the autopilot activation switch, and the NWC computer predicted oscillation frequency for a mode similar to the classical first tangential, is shown in Figure 6 (provided by Bill Caywood at the 18 November 1975 APL meeting on Mk 12). There is good correlation between the predicted and measured frequencies.

The slight differences are explained by the fact that the ratio of propellant combustion products changes throughout burn but the ratio was assumed to be constant in the calculations. With this degree of correlation it appears highly probable that the vibrations experienced in the Mk 12 during flight and static tests are generated by combustion instability.

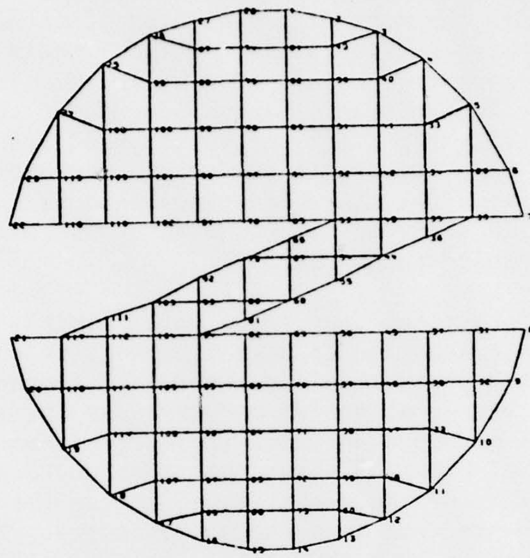

FIGURE 1. Finite Element Grid Used for Computer-Prediction of Acoustics in the Mk 12 Motor. The grid shown represents 25% web burn.

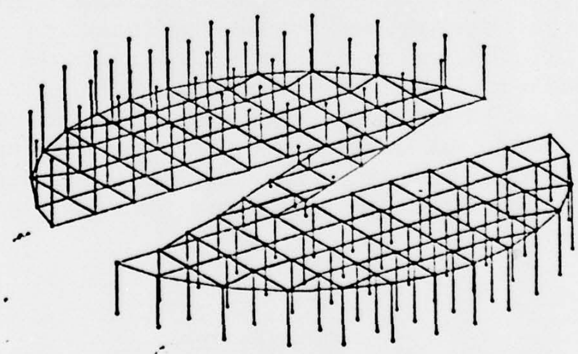

FIGURE 2. Computer-Predicted Coupled-Cavities Mode for Mk 12 Motor at 25% Web Burn: 608 Hz.



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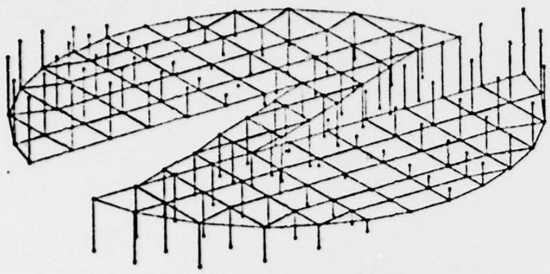
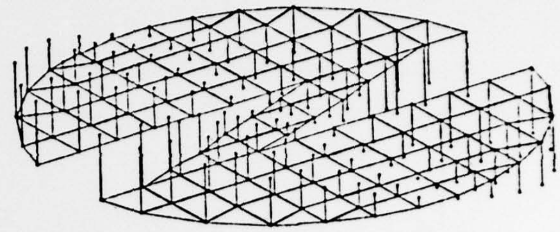
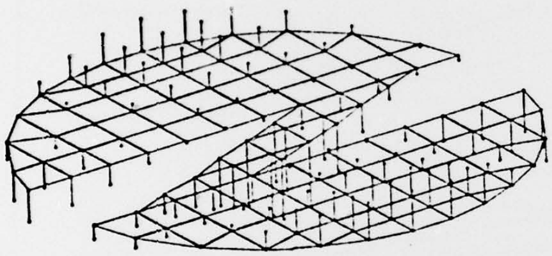


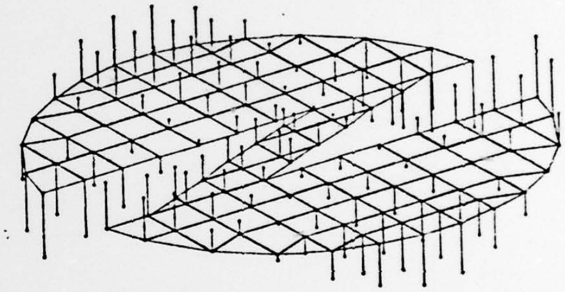
(a) First Tangential (abnormal)
1,960 Hz.



(b) First Tangential (normal)
2,102 Hz.



(c) Section Tangential (normal)
3,454 Hz.



(d) Second Tangential (abnormal)
3,578 Hz.

FIGURE 3. Computer-Predicted Wave Structure and Frequency for Tangential Modes in the Mk 12 Motor at 25% Web Burn.

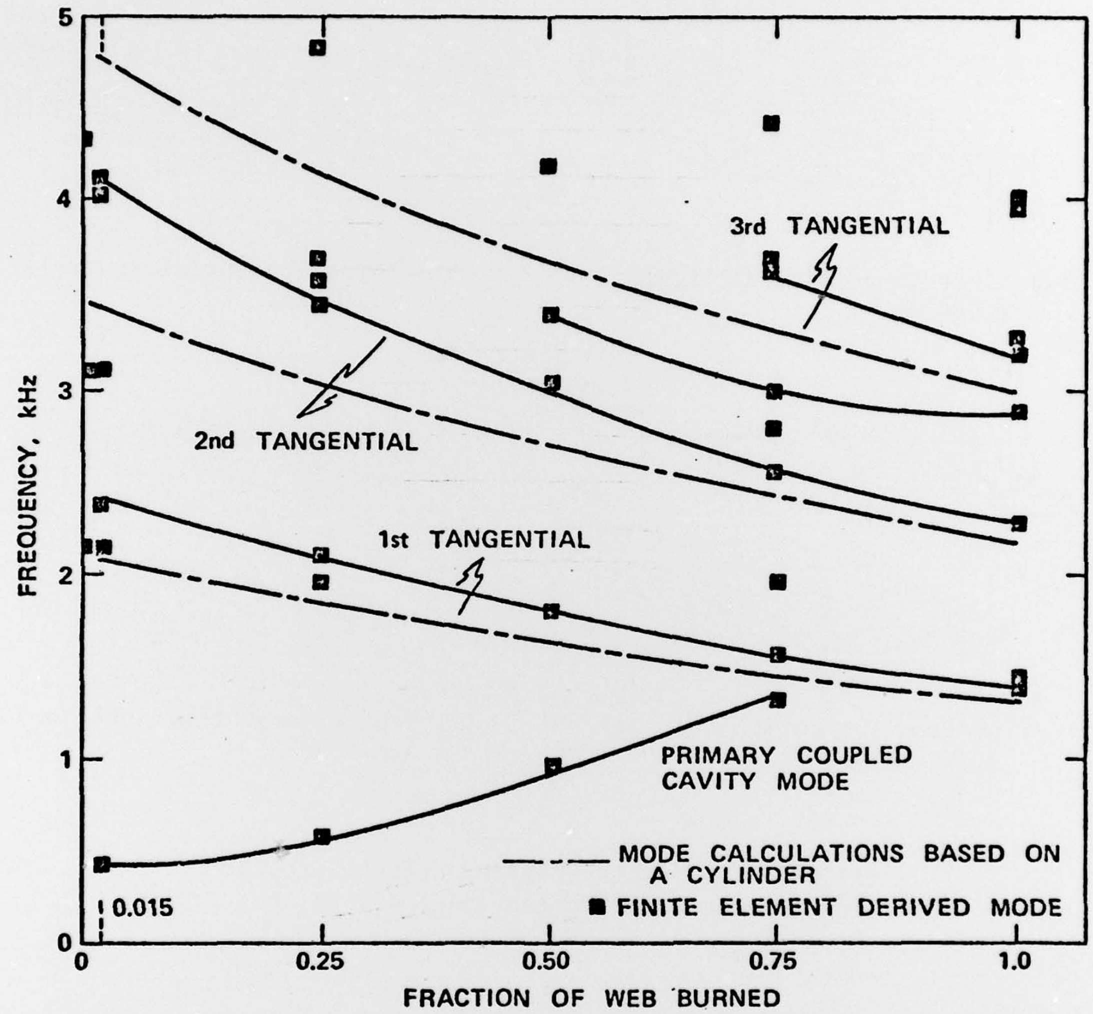
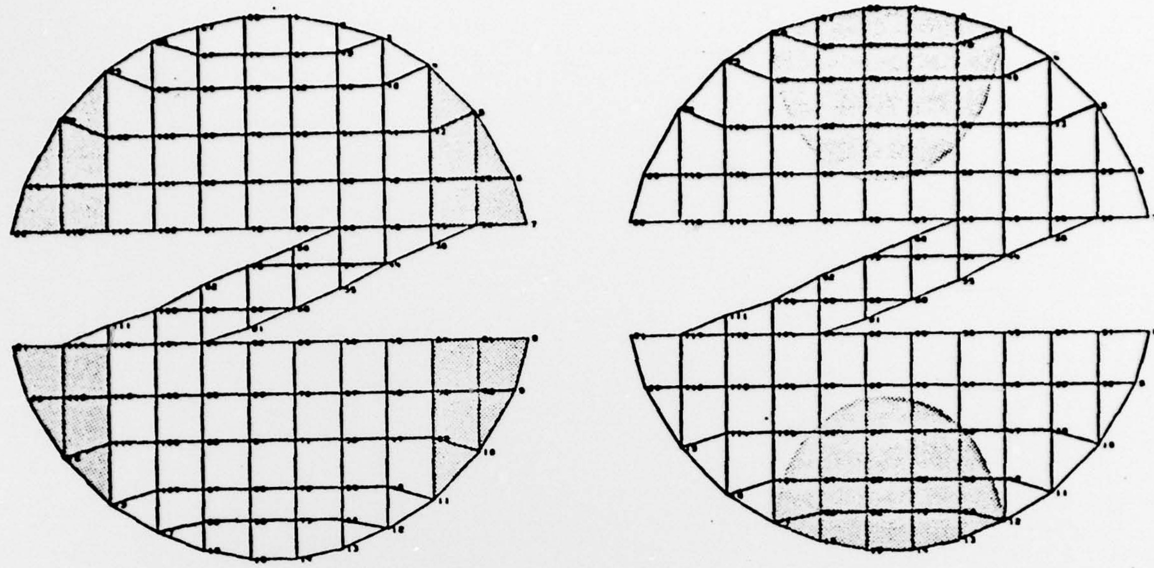



FIGURE 4. Frequency-Time Plot Showing Comparison of Hand and Computer-Predicted Tangential Acoustic Frequencies.



(a) Optimum Areas (shaded) (b) Non-Optimum Areas (shaded)

FIGURE 5. Optimum and Non-Optimum Areas for Pressure Transducer Location.

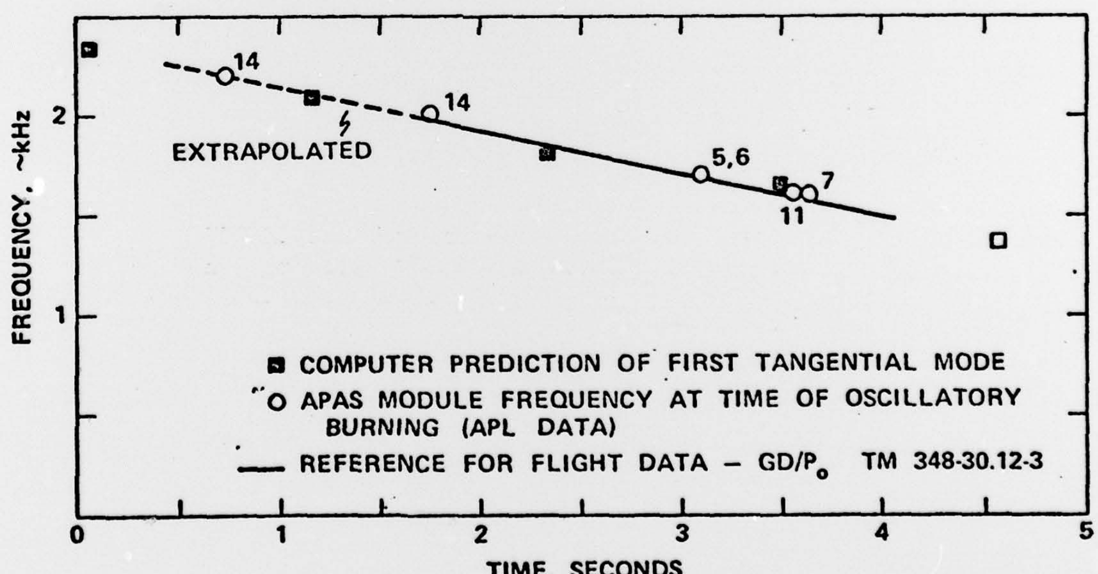



FIGURE 6. Comparison of Computer-Predicted Acoustic Frequency and Experimental Frequency Measurements From Analysis of Accelerometer Data on Mk 12 Motor Firings. (Numbers in the Figure represent APL test numbering system.)

Appendix A
USE OF CLASSICAL ACOUSTICS OF A RIGHT CIRCULAR CYLINDER
TO ESTIMATE MK 12 MOTOR FREQUENCIES

Acoustic oscillations in a fluid medium are pressure oscillations of small amplitude and are described mathematically by the classical wave equation. For a cylindrical cavity with closed ends and ideally rigid walls the acoustic pressure variation can be calculated using:

$$\hat{P}_{m,n,n_z} = \sum\left[J_m\left(\frac{\pi\alpha_{mn}r}{R}\right)\right]\cdot\cos\left(\frac{n_z\pi z}{L}\right) \tag{A-1}$$

$$\cdot\left[A_1\cos(m\phi - \omega t - S_1) + A_2\cos(m\phi - \omega t - S_2)\right]$$ (Eq. 3 of footnote 3)

in which

| | |
|---|---|
| $\hat{P}$ | is the difference between local and space averaged pressure at any point in space and time |
| $r,\phi,z$ | Are the cylindrical coordinates with the origin at the center of one end of the cavity |
| $R,L$ | Radius and length of the cavity |
| $m,n,n_z$ | Wave numbers characterizing any particular mode of oscillation |
| $J_m$ | Bessel function of order m |
| $\alpha_{mn}$ | nth root of the equation $\frac{d}{dx}J_m(\pi x) = 0$ (Some values are given in Table A-1) |
| $A_1,A_2$ | Arbitrary independent amplitude constants |
| $S_1,S_2$ | Arbitrary independent phase constants |
| $t$ | Time |
| $\omega$ | Circular frequency |

Every possible acoustic mode has its frequency which, for a cylindrical cavity, can be calculated using the following equation:

$$f_{m,n,n_z} = C/2\left[\left(\frac{\alpha_{mn}}{R}\right)^2 + \left(\frac{n_z}{L}\right)^2\right]^{1/2} \tag{A-2}$$

TABLE A-1. Values of $\alpha_{mn}$

| Tangential wave No., m | Radial wave number, n | | | |
|---|---|---|---|---|
| | 0 | 1 | 2 | 3 |
| 0 | 0.000 | 1.220 | 2.233 | 3.238 |
| 1 | 0.586 | 1.697 | 2.714 | 3.726 |
| 2 | 0.972 | 2.135 | 3.173 | 4.192 |
| 3 | 1.337 | 2.551 | 3.611 | 4.643 |

Any particular mode of oscillation is identified by the wave number in each of the three directions, axial ($n_z$), radial (n), and tangential (m). Values of $\alpha_{mn}$ for wave numbers up to 3 are given in Table A-1. Where only one wave number is not zero, the corresponding mode is a pure mode. Pure modes with $m \neq 0$ are tangential, thus the first tangential is $m = 1$, $n = 0$, $n_z = 0$. Figure A-1 shows the pressure distribution for tangential waves of order 1, 2, and 3.

Rough estimates of tangential acoustic frequencies to be found in the Mk 12 were calculated using Eq. (A-2), values of m = 1, 2, and 3, n = 0, $n_z = 0$, and c = 3,250 ft/s (9.9 x $10^4$ cm/s). The initial value of R was set at 5.44 in. (13.8 cm) and its final value was 8.70 in. (22.2 cm). No allowance was made in these calculations for the effect of the protruding wedges of propellant which are present in the Mk 12 grain design. The hand calculations, using the above data, provide a table of frequencies for the three tangential modes and for various times during burn as shown in Table A-2. These data were used to establish the dashed curves in Figure 4 for comparison with the 2-dimensional NASTRAN computer prediction of frequency.

TABLE A-2. Tangential Wave Frequencies (Hz) for Mk 12 Motor (Hand Calculations)

| Tangential wave number | Percent of web burned | | | | |
|---|---|---|---|---|---|
| | 0 | 25 | 50 | 75 | 100 |
| 1 | 2,103 | 1,828 | 1,616 | 1,449 | 1,313 |
| 2 | 3,488 | 3,033 | 2,681 | 2,404 | 2,178 |
| 3 | 4,797 | 4,171 | 3,687 | 3,306 | 2,996 |

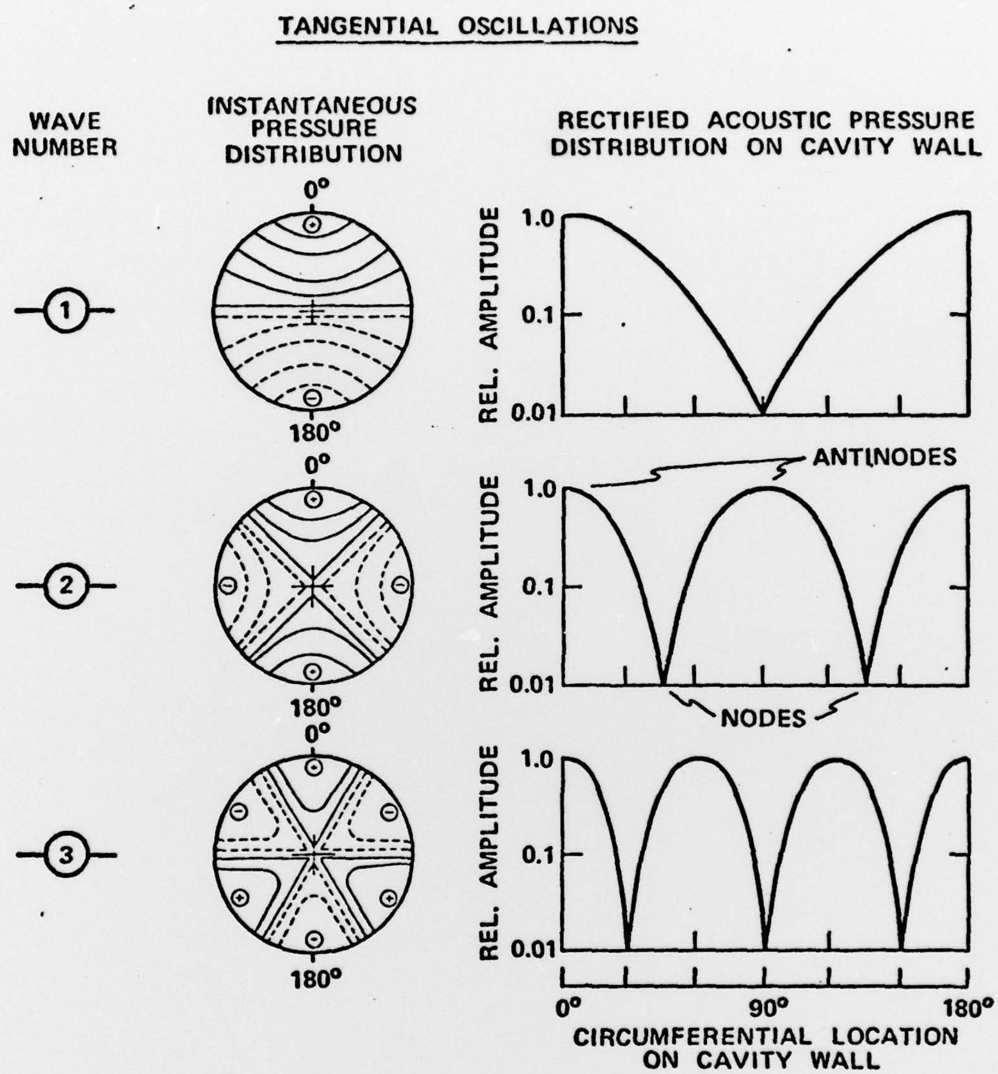



FIGURE A-1. Tangential Wave Acoustic Pressure Distributions in a Right Circular Cylindrical Cavity.